MA 3046

Matrix Analysis

Exam I - Quarter IV, AY 01-02

Instructions: Work all problems. Show appropriate intermediate computations for full credit. Calculators and one page of notes ($8\frac{1}{2}$ by 11 inches, both sides) permitted. *Read the questions carefully.*

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1. (30 points) Consider the set of vectors

$$\mathbf{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Use the classical Gram-Schmidt process to convert these to an **orthonormal** set.

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2. (30 points) Consider the following problem:

$$\mathbf{Q}\,\mathbf{R}\,\mathbf{x} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 8 \\ 8 \\ 16 \\ -8 \end{bmatrix}$$

a. Using the fact that $\mathbf{Q}$ is a unitary matrix, solve this system.

b. Does your solution actually satisfy the given equation? If not, is there any possible reason, other than an algebra error, why that could have occurred? (*Briefly* explain your answer.)

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3. (15 points) Use the singular value decomposition to show that the singular values of $\mathbf{A}^H\,\mathbf{A}$ are precisely the squares of the singular values of $\mathbf{A}$

Additional Problem On Reverse Side!

4. (25 points) a. Assume that matrix $\mathbf{A}$ has condition number $\kappa(\mathbf{A}) = 10^6$. In a ten decimal digit computer, about many accurate significant digits can be expected in the solution to

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

if iterative improvement is **not** used?

b. Identify two factors, beside the number of floating-point operations (flops) required and the CPU speed, that determine the total time required to execute a computational algorithm.

c. A 500×500 matrix $\mathbf{A}$ must undergo a rank one update given by:

$$\mathbf{A} - \mathbf{v}\,\mathbf{v}^H$$

where $\mathbf{v}$ is a 500×1 vector, the first four hundred fifty elements of which are *identically zero*. The result will be stored in the location occupied by the original matrix. Give no more than four lines of MATLAB code that will accomplish this in a highly efficient manner.
